

Questions

20N.1.SL.TZ0.T_6

Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

a.i. Find the probability that the first ball chosen is labelled A. [1]

a.ii. Find the probability that the first ball chosen is labelled A or labelled N. [1]

b.

Find the probability that the second ball chosen is labelled A, given that the first ball chosen was labelled N.

[2]

c. Find the probability that both balls chosen are labelled N. [2]

19M.2.SL.TZ1.S_10

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

Ted plays a game using these dice. The rules are:

Having a turn means to roll all three dice.

He wins \$10 for each green face rolled and adds this to his winnings.

After a turn Ted can either:

end the game (and keep his winnings), or

have another turn (and try to increase his winnings).

If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

The random variable D (\$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D , where $\$w$ represents his winnings in the game so far.

D (\$)	$-w$	0	10	20	30
$P(D = d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

a.i. Find the probability of rolling exactly one red face. [2]

a.ii. Find the probability of rolling two or more red faces. [3]

b.

Show that, after a turn, the probability that Ted adds exactly \$10 to his winnings is $\frac{1}{3}$.

[5]

c.i. Write down the value of x . [1]

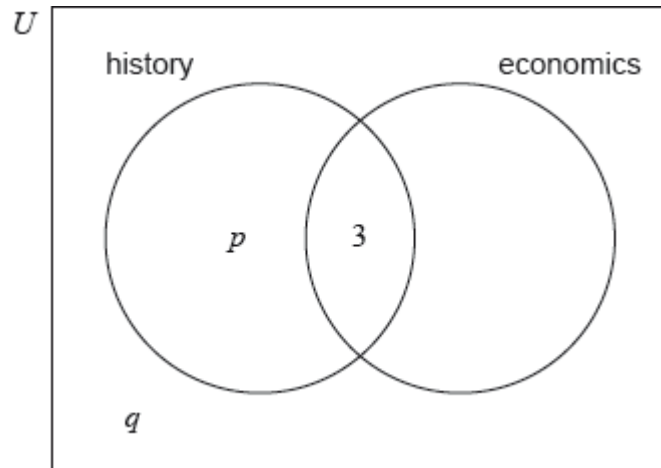
c.ii. Hence, find the value of y . [2]

d. Ted will always have another turn if he expects an increase to his winnings.

Find the least value of w for which Ted should end the game instead of having another turn.

[3]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



a.i. Find the value of p ; [2]

a.ii. Find the value of q . [2]

b.

A girl is selected at random. Find the probability that she takes economics but not history.

[2]

18N.1.SL.TZ0.S_9

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

Let $n = 5$. Find the probability that the game will end on her

a.i. Find the probability, in terms of n , that the game will end on her first draw. [1]

a.ii. Find the probability, in terms of n , that the game will end on her second draw.

[3]

b.i. third draw. [2]

b.ii. fourth draw. [2]

c.

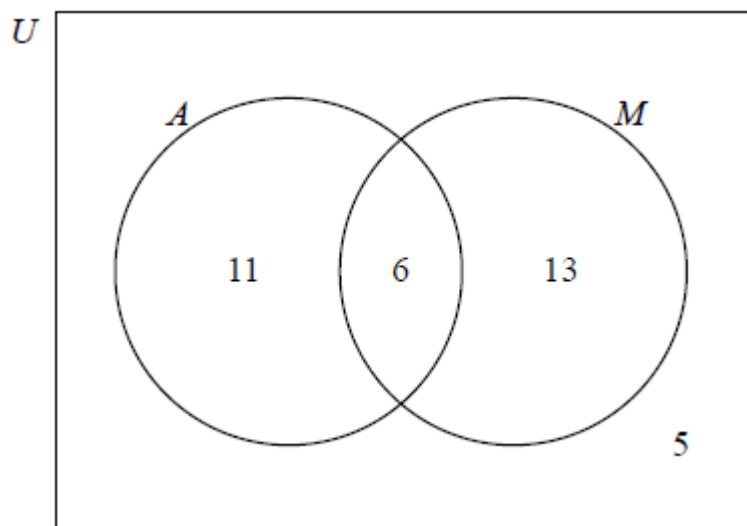
Hayley plays the game when $n = 5$. She pays \$20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

Number of draws	1	2	3	4
Money earned back (\$ M)	0	20	$8k$	$12k$

Find the value of k so that this is a fair game. [7]

18N.2.SL.TZ0.S_1

In a group of 35 students, some take art class (A) and some take music class (M). 5 of these students do not take either class. This information is shown in the following Venn diagram.



One student from the group is chosen at random. Find the probability that

- a. Write down the number of students in the group who take art class. [2]
- b.i. the student does not take art class. [2]
- b.ii. the student takes either art class or music class, but not both. [2]

22M.1.SL.TZ2.9

A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $E(X) = 2$.

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled 1, 2, 3 and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled 1, 2, 2 and 3. The second red die has faces labelled 1, a , a and b , where $a < b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

a. Show that $p = 0.4$ and $q = 0.2$. [5]

b. Find $P(X > 2)$. [2]

c.

Assuming that rolls of the die are independent, find the probability that Nicky wins the game.

[5]

d. Determine the value of b . [2]

e. Find the value of a , providing evidence for your answer. [2]

Markschemes

20N.1.SL.TZ0.T_6

a.i.

$\frac{3}{9}$ $\frac{1}{3}$, 0.333, 0.333333..., 33.3% (A1) (C1)

[1 mark]

a.ii. $\frac{5}{9}$ 0.556, 0.555555..., 55.6% (A1) (C1)

[1 mark]

b. $\frac{3}{8}$ 0.375, 37.5% (A1)(A1) (C2)

Note: Award (A1) for correct numerator, (A1) for correct denominator. [2 marks]

c. $\frac{2}{9} \times \frac{1}{8}$ (M1)

Note: Award (M1) for a correct compound probability calculation seen.

$\frac{2}{72}$ $\frac{1}{36}$, 0.0278, 0.0277777..., 2.78% (A1) (C2) [2 marks]

19M.2.SL.TZ1.S_10

a.i.

valid approach to find $P(\text{one red})$ (M1)

$$\text{eg } {}_n C_a \times p^a \times q^{n-a}, B(n, p), 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2, \binom{3}{1}$$

listing all possible cases for exactly one red (may be indicated on tree diagram)

$$P(1 \text{ red}) = 0.444 \left(= \frac{4}{9} \right) \quad [0.444, 0.445] \quad \mathbf{A1 \ N2}$$

[3 marks] [5 maximum for parts (a.i) and (a.ii)]a.ii. valid approach (M1) eg $P(X=2) + P(X=3)$, $1 - P(X \leq 1)$, $\text{binomcdf}\left(3, \frac{1}{3}, 2, 3\right)$

$$\text{correct working (A1) eg } \frac{2}{9} + \frac{1}{27}, 0.222 + 0.037, 1 - \left(\frac{2}{3}\right)^3 - \frac{4}{9} \quad 0.259259$$

$$P(\text{at least two red}) = 0.259 \left(= \frac{7}{27} \right) \quad \mathbf{A1 \ N3}$$

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

b. recognition that winning \$10 means rolling exactly one green (M1)

recognition that winning \$10 also means rolling at most 1 red (M1)

eg "cannot have 2 or more reds" correct approach A1

$$\text{eg } P(1G \cap 0R) + P(1G \cap 1R), P(1G) - P(1G \cap 2R),$$

"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this A1 for purely numerical expressions.

one correct probability for their approach (A1)

$$\text{eg } 3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2, \frac{6}{27}, 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2, \frac{1}{9}, \frac{2}{9} \quad \text{correct working leading to } \frac{1}{3} \quad \mathbf{A1}$$

$$\text{eg } \frac{3}{27} + \frac{6}{27}, \frac{12}{27} - \frac{3}{27}, \frac{1}{9} + \frac{2}{9} \quad \text{probability} = \frac{1}{3} \quad \mathbf{AG \ N0 \ [5 marks]}$$

$$\text{c.i. } x = \frac{7}{27}, 0.259 \text{ (check FT from (a)(ii))} \quad \mathbf{A1 \ N1 \ [1 mark]}$$

c.ii. evidence of summing probabilities to 1 (M1)

$$\text{eg } \Sigma = 1, x + y + \frac{1}{3} + \frac{2}{9} + \frac{1}{27} = 1, 1 - \frac{7}{27} - \frac{9}{27} - \frac{6}{27} - \frac{1}{27}$$

$$0.148147 \text{ (0.148407 if working with their } x \text{ value to 3 sf)}$$

$$y = \frac{4}{27} \text{ (exact), } 0.148 \quad \mathbf{A1 \ N2 \ [2 \ marks]}$$

d. correct substitution into the formula for expected value **(A1)**

$$\text{eg } -w \cdot \frac{7}{27} + 10 \cdot \frac{9}{27} + 20 \cdot \frac{6}{27} + 30 \cdot \frac{1}{27} \quad \text{correct critical value (accept inequality)} \quad \mathbf{A1}$$

$$\text{eg } w = 34.2857 \left(= \frac{240}{7} \right), \quad w > 34.2857 \quad \$40 \quad \mathbf{A1 \ N2 \ [3 \ marks]}$$

17M.1.SL.TZ1.S_1

a.i.

valid approach **(M1)**

$$\text{eg } p + 3 = 13, 13 - 3$$

$$p = 10 \quad \mathbf{A1 \ N2}$$

[2 marks]

a.ii. valid approach **(M1)** eg $p + 3 + 5 + q = 20, 10 - 10 - 8 \quad q = 2 \quad \mathbf{A1 \ N2}$

[2 marks]

b. valid approach **(M1)** eg $20 - p - q - 3, 1 - \frac{15}{20}, n(E \cap H') = 5$

$$\frac{5}{20} \left(\frac{1}{4} \right) \quad \mathbf{A1 \ N2 \ [2 \ marks]}$$

18N.1.SL.TZ0.S_9

a.i.

$$\frac{2}{n} \quad \mathbf{A1 \ N1}$$

[1 mark]a.ii. correct probability for one of the draws **A1**

$$\text{eg } P(\text{not blue first}) = \frac{n-2}{n}, \text{ blue second} = \frac{2}{n-1} \quad \text{valid approach} \quad \mathbf{(M1)}$$

eg recognizing loss on first in order to win on second, $P(B' \text{ then } B)$, $P(B') \times P(B | B')$, tree diagram

$$\text{correct expression in terms of } n \quad \mathbf{A1 \ N3} \quad \text{eg } \frac{n-2}{n} \times \frac{2}{n-1}, \frac{2n-4}{n^2-n}, \frac{2(n-2)}{n(n-1)}$$

[3 marks]

$$\text{b.i. correct working} \quad \mathbf{(A1)} \quad \text{eg } \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{12}{60} \left(= \frac{1}{5} \right) \quad \mathbf{A1 \ N2} \quad \mathbf{[2 marks]}$$

$$\text{b.ii. correct working} \quad \mathbf{(A1)} \quad \text{eg } \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} = \frac{6}{60} \left(= \frac{1}{10} \right) \quad \mathbf{A1 \ N2} \quad \mathbf{[2 marks]}$$

c. correct probabilities (seen anywhere) **(A1)(A1)**

$$\text{eg } P(1) = \frac{2}{5}, P(2) = \frac{6}{20} \quad (\text{may be seen on tree diagram})$$

valid approach to find $E(M)$ or expected winnings using **their** probabilities **(M1)**

$$\text{eg } P(1) \times (0) + P(2) \times (20) + P(3) \times (8k) + P(4) \times (12k),$$

$$P(1) \times (-20) + P(2) \times (0) + P(3) \times (8k - 20) + P(4) \times (12k - 20)$$

correct working to find $E(M)$ or expected winnings **(A1)**

$$\text{eg } \frac{2}{5}(0) + \frac{3}{10}(20) + \frac{1}{5}(8k) + \frac{1}{10}(12k),$$

$$\frac{2}{5}(-20) + \frac{3}{10}(0) + \frac{1}{5}(8k - 20) + \frac{1}{10}(12k - 20)$$

correct equation for fair game **A1**

$$\text{eg } \frac{3}{10}(20) + \frac{1}{5}(8k) + \frac{1}{10}(12k) = 20, \frac{2}{5}(-20) + \frac{1}{5}(8k - 20) + \frac{1}{10}(12k - 20) = 0$$

correct working to combine terms in k **(A1)**

$$\text{eg } -8 + \frac{14}{5}k - 4 - 2 = 0, 6 + \frac{14}{5}k = 20, \frac{14}{5}k = 14 \quad k = 5 \quad \mathbf{A1 \ NO}$$

Note: Do not award the final **A1** if the candidate's **FT** probabilities do not sum to 1.

[7 marks]

18N.2.SL.TZ0.S_1

a.

** This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.*

valid approach **(M1)**

$$(A \cap M') + (A \cap M), \frac{17}{35}, 11 + 6$$

number of students taking art class = 17 **A1 N2**

[2 marks]

b.i. valid approach **(M1)** $13 + 5, 35 - 17, 18, 1 - P(A)$ 0.514285

$$P(A) = \frac{18}{35} \text{ (exact), } 0.514 \quad \mathbf{A1 N2} \quad \mathbf{[2 marks]}$$

b.ii. valid approach **(M1)** $11 + 13, 35 - 6 - 5, 24$ 0.685714

$$P(A \text{ or } M \text{ but not both}) = \frac{24}{35} \text{ (exact), } 0.686 \quad \mathbf{A1 N2} \quad \mathbf{[2 marks]}$$

22M.1.SL.TZ2.9

a.

uses $\sum PX = x = 1$ to form a linear equation in p and q (M1)correct equation in terms of p and q from summing to 1 A1

$$p + 0.3 + q + 0.1 = 1 \quad \text{OR} \quad p + q = 0.6 \quad (\text{or equivalent})$$

uses $EX = 2$ to form a linear equation in p and q (M1)correct equation in terms of p and q from $EX = 2$ A1

$$p + 0.6 + 3q + 0.4 = 2 \quad \text{OR} \quad p + 3q = 1 \quad (\text{or equivalent})$$

Note: The marks for using $\sum PX = x = 1$ and the marks for using $EX = 2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously A1

$$\text{for example, } 2q = 0.4 \Rightarrow q = 0.2 \quad \text{or} \quad p + 3 \times 0.6 - p = 1 \Rightarrow p = 0.4$$

$$\text{so } p = 0.4 \text{ and } q = 0.2 \quad \mathbf{AG}$$

[5 marks]

b. valid approach (M1)

$$P(X > 2) = P(X = 3) + P(X = 4) \quad \text{OR} \quad P(X > 2) = 1 - P(X = 1) - P(X = 2)$$

$$= 0.3 \quad \mathbf{A1} \quad \mathbf{[2 marks]}$$

c.

recognises at least one of the valid scores (6, 7, or 8) required to win the game (M1)

Note: Award **M0** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let T represent the score on the last two rolls

a score of 6 is obtained by rolling 2, 4, 4, 2 or 3, 3

$$PT = 6 = 20 \cdot 30 \cdot 1 + 0.2^2 = 0.1 \quad \mathbf{A1}$$

$$\text{a score of 7 is obtained by rolling 3, 4 or 4, 3} \quad PT = 7 = 20 \cdot 20 \cdot 1 = 0.04 \quad \mathbf{A1}$$

a score of 8 is obtained by rolling 4, 4 $PT = 8 = 0.1^2 = 0.01$ **A1**

Note: The above 3 **A1** marks are independent of each other.

$P(\text{Nicky wins}) = 0.1 + 0.04 + 0.01 = 0.15$ **A1** **[5 marks]**

d. $3 + b = 8$ **(M1)** $b = 5$ **A1** **[2 marks]**

e. **METHOD 1 EITHER** $PS = 5 = \frac{4}{16}$ $PS = a + 2 = \frac{4}{16}$ **A1** $\Rightarrow a + 2 = 5$ **OR**

$PS = 6 = \frac{3}{16}$ $PS = a + 3 = \frac{2}{16}$ and $PS = 5 + 1 = \frac{1}{16}$ **A1** $\Rightarrow a + 3 = 6$ **OR**

$PS = 4 = \frac{3}{16}$ $PS = a + 1 = \frac{2}{16}$ and $PS = 1 + 3 = \frac{1}{16}$ **A1** $\Rightarrow a + 1 = 4$ **THEN**

$\Rightarrow a = 3$ **A1**

Note: Award **A0A0** for $a = 3$ obtained without working/reasoning/justification.

METHOD 2 EITHER correctly lists a relevant part of the sample space **A1**

for example, $S = 4 = 3, 1, 1, a, 1, a$ or $S = 5 = 2, a, 2, a, 2, a, 2, a$ or $S = 6 = 3, a, 3, a, 1, 5$

$a + 3 = 6$ eliminates possibilities (exhaustion) for $a < 5$ **OR**

convincingly shows that $a \neq 2, 4$ **A1**

$a \neq 4$, for example, $PS = 7 = \frac{2}{16}$ from 2, 5, 2, 5 and so $3, a, 3, a \Rightarrow a + 3 \neq 7$ **THEN**

$\Rightarrow a = 3$ **A1** **[2 marks]**